take note

Key Concept Relative Frequency

Relative frequency is the ratio of the frequency of the category to the total frequency.

$$\text{relative frequency} = \frac{\text{frequency of rock music preference}}{\text{total frequency}}$$

$$= \frac{10}{10+7+8+5+6+4} = \frac{10}{40} = \frac{1}{4}$$

The relative frequency of preference for rock music is $\frac{1}{4}$.

Type of Music Preferred	Frequency
Rock	10
Hip Hop	7
Country	8
Classical	5
Alternative	6
Other	4

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A student conducts a probability experiment by tossing 3 coins one after the other. Using the results below, what is the probability that exactly two heads will occur in the next three tosses?

Coin Toss Result	HHH	HHT	HTT	HTH	THH	THT	TTT	TTH
Frequency	5	7	9	6	2	9	10	2

$$\text{relative frequency} = \frac{\text{frequency of exactly two heads}}{\text{total of the frequencies}} = \frac{15}{50} = \frac{3}{10}$$

Sample space is the total number of possible outcomes.

In this example there are 8 possible outocomes.

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Key Concept Fundamental Counting Principle

The **Fundamental Counting Principle** says that if event M occurs in m ways and event N occurs in n ways, then event M followed by event N can occur in $m \cdot n$ ways.

Example 4 entrees and 6 drinks gives $4 \cdot 6 = 24$ possible lunch specials

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Example: suppose that you are choosing an outfit by first selecting pants and then a shirt. If there are two jean choices and three shirt choices then m * n is 2 * 3 or 6.

Pants	Shirts	Outfits
blue jeans	red	jeans, red shirt
	green	jeans, green shirt
	blue	jeans, blue shirt
black pants	red	pants, red shirt
	green	pants, green shirt
	blue	pants, blue shirt

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There are 6 possible sandwiches, 4 different side items, and 5 drink choices.

$6 \cdot 4 \cdot 5 = 120$ Use the Fundamental Counting Principle.

There are 120 different possible lunch specials.

Q:If you do not select a drink how many combinations are there?

A: 6 * 4 = 24

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A **permutation** is an arrangement of items in which the order of the objects is important. You can use the Fundamental Counting Principle to find the total number of permutations. Suppose you want to find the number of ways to line up 4 friends. There are 4 ways to choose the first person, 3 ways to choose the second person, 2 ways to choose the third, and only 1 way to choose the last. So, there are $4 \cdot 3 \cdot 2 \cdot 1 = 24$ permutations, or ways to arrange your friends.

You can use *factorial notation* to write $4 \cdot 3 \cdot 2 \cdot 1$ as 4!

For any positive integer n, **n factorial** is $n!$

Note: 0! = 1

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Take note

Key Concept Permutation Notation

The number of permutations of n items of a set arranged r items at a time is

$$_nP_r = \frac{n!}{(n-r)!} \text{ for } 0 \le r \le n.$$

Example $_8P_3 = \frac{8!}{(8-3)!} = \frac{8!}{5!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 8 \cdot 7 \cdot 6 = 336$

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Problem 3 Finding a Permutation

The environmental club is electing a president, vice president, and treasurer. How many different ways can the officers be chosen from the 10 members?

Use the formula for permutations.

There are 10 members, arranged 3 at a time. So $n = 10$ and $r = 3$.

$_{10}P_3 = \frac{10!}{(10-3)!}$ Substitute 10 for n and 3 for r.

$= \frac{3{,}628{,}800}{5040} = 720$ Simplify.

Find the ! key on your calculator to avoid having to press 10*9*8*7*6*5*4*3*2*1.

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take note

Key Concept Combination Notation

The number of combinations of n items chosen r at a time is

$$_nC_r = \frac{n!}{r!(n-r)!} \text{ for } 0 \leq r \leq n.$$

Example $$_9C_4 = \frac{9!}{4!(9-4)!} = \frac{9!}{4!\,5!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(4 \cdot 3 \cdot 2 \cdot 1)(5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)} = \frac{9 \cdot 8 \cdot 7 \cdot 6}{4 \cdot 3 \cdot 2 \cdot 1} = 126$$

A **combination** is a selection of items in which order is *not* important. Suppose you select 3 different fruits to make a fruit salad. The order you select the fruits does not matter.

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Problem 5 Identifying Combinations and Permutations

A **A college student is choosing 3 classes to take during first, second, and third semseter from the 5 elective classes offered in his major. How many possible ways can the student schedule the three classes?**

The order in which the classes are chosen does matter. Use a permutation.

$$_5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = 60$$

There are 60 ways that the student can schedule the three classes.

B **A jury of 12 people is chosen from a pool of 35 potential jurors. How many different juries can be chosen?**

The order in which the jurors are chosen is not important. Use a combination.

$$_{35}C_{12} = \frac{35!}{12!(35-12)!} = \frac{35!}{12!\,23!} = 834{,}451{,}800$$

There are 834,451,800 possible juries of 12 people.

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Problem 6 Finding Probabilities

Three pool balls are randomly chosen from a set numbered from 1 to 15. What is the probability that the pool balls chosen are numbered 5, 7, and 9?

Step 1 Use the probability formula.

$$P(\text{choosing 5, 7, and 9}) = \frac{\text{number of possible ways to choose 5, 7, and 9}}{\text{number of ways to choose 3 pool balls}}$$

Step 2 Find the numerator. Use the Fundamental Counting Principle to find the number of possible ways to choose 5, 7, and 9.

There are 3 ways to pick the first ball, 2 ways to pick the second ball, and 1 way to pick the last ball.

$$3 \cdot 2 \cdot 1 = 6$$

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Step 3 Find the sample space. Because choosing pool balls numbered 5, 7, and 9 is the same outcome as choosing pool balls numbered 9, 5, and 7, the order does not matter. Use the combination formula to find the total number of ways to choose 3 pool balls from 15 pool balls.

$$_{15}C_3 = \frac{15}{3!(15-3)!} = 455$$

Step 4 Find the probability.

$$P(\text{choosing 5, 7, and 9}) = \frac{6}{455} \approx 0.013$$

The probability of choosing the pool balls numbered 5, 7, and 9 is about 0.013, or 1.3%.

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Key Concept Calculating Expected Value

If A is an action that includes outcomes $A_1, A_2, A_3, \ldots$ and Value(A_n) is a quantitative value associated with each outcome, the expected value of A is given by

$$\text{Value}(A) = P(A_1) \cdot \text{Value}(A_1) + P(A_2) \cdot \text{Value}(A_2) + \ldots$$

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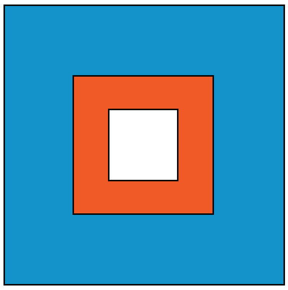
Suppose you are at a carnival and are throwing darts at a board like the one at the right. There is an equally likely chance that your dart lands anywhere on the board. You receive 20 points if your dart lands in the white area, 10 points if it lands in the red area, and -5 points if it lands in the blue area. How many points can you expect to get given that the areas for each region are white, 36 in.^{2}; red, 108 in.^{2}; and blue, 432 in.^{2}? The total area is 576 in.^{2}.



Value(throw)

$= P(\text{white area}) \cdot (\text{white points}) + P(\text{red area}) \cdot (\text{red points}) + P(\text{blue area}) \cdot (\text{blue points})$

$= \frac{36}{576} \cdot 20 + \frac{108}{576} \cdot 10 + \frac{432}{576} \cdot (-5)$

$= 1.25 + 1.875 + (-3.75)$

$= -0.625$

You can expect to get -0.625 points.

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Problem 1 Making Random Selections

There are 28 students in a homeroom. Four students are chosen at random to represent the homeroom on a student committee. How can a random number table be used to fairly choose the students?

Step 1 Select a line from a random number table. <-this is usually given to you

18823 18160 93593 67294 09632 62617 86779

Step 2 Group the line from the table into two digit numbers.

18 82 31 81 60 93 59 36 72 94 09 63 26 26 17 86 77 9

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Step 3 Match the first four numbers less than 28 with the position of the students' names on a list. Duplicates and numbers greater than 28 are discarded because they don't correspond to any student on the list.

18 82 31 81 60 93 59 36 72 94 09 63 26 26 17 86 77 9

The students listed 18th, 9th, 26th, and 17th on the list are chosen fairly.

Note: you can use your calculator/phone to generate random numbers as well!

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